Scheda di lavoro: formule goniometriche

Formule di addizione e sottrazione:

$$cos(\alpha \pm \beta) = cos\,\alpha \cdot cos\,\beta \mp sen\,\alpha \cdot sen\,\beta$$

$$sen(\alpha \pm \beta) = sen\,\alpha \cdot cos\,\beta \pm cos\,\alpha \cdot sen\,\beta$$

Formule di duplicazione:

$$sen\,2\alpha = sen(\alpha + \alpha) = 2sen\,\alpha \cdot cos\,\alpha$$

$$cos\,2\alpha = cos(\alpha + \alpha) = cos^2\alpha - sen^2\alpha$$

Formule di bisezione:

$$cos\,2\beta = cos^2\beta - sen^2\beta = 1 - sen^2\beta - sen^2\beta = 1 - 2sen^2\beta \quad \rightarrow \quad \boxed{sen^2\beta = \frac{1 - cos\,2\beta}{2}}$$

$$\rightarrow \; sen\,\beta = \pm\sqrt{\frac{1 - cos2\beta}{2}} \qquad \rightarrow^* \; sen\frac{\alpha}{2} = \pm\sqrt{\frac{1 - cos\alpha}{2}}$$

$$cos\,2\beta = cos^2\beta - sen^2\beta = cos^2\beta - (1 - cos^2\beta) = 2cos^2\beta - 1 \quad \rightarrow \quad \boxed{cos^2\beta = \frac{1 + cos\,2\beta}{2}}$$

$$\rightarrow \; cos\,\beta = \pm\sqrt{\frac{1 + cos\,2\beta}{2}} \qquad \rightarrow^* \; cos\frac{\alpha}{2} = \pm\sqrt{\frac{1 + cos\alpha}{2}}$$

Formule parametriche:

$$sen\,2\beta = \frac{2sen\beta \cdot cos\,\beta}{1} = \frac{2sen\,\beta \cdot cos\,\beta}{cos^2\beta + sen^2\beta} = \frac{\dfrac{2sen\,\beta \cdot \cancel{cos\,\beta}}{cos^{\cancel{2}}\beta}}{\dfrac{cos^2\beta}{cos^2\beta} + \dfrac{sen^2\beta}{cos^2\beta}} = \frac{2tg\,\beta}{1 + tg^2\beta} \qquad \rightarrow^* \; sen\,\alpha = \frac{2tg\dfrac{\alpha}{2}}{1 + tg^2\dfrac{\alpha}{2}}$$

$$cos\,2\beta = \frac{cos^2\beta - sen^2\beta}{1} = \frac{cos^2\beta - sen^2\beta}{cos^2\beta + sen^2\beta} = \frac{\dfrac{cos^2\beta}{cos^2\beta} - \dfrac{sen^2\beta}{cos^2\beta}}{\dfrac{cos^2\beta}{cos^2\beta} + \dfrac{sen^2\beta}{cos^2\beta}} = \frac{1 - tg^2\beta}{1 + tg^2\beta} \qquad \rightarrow^* \; cos\,\alpha = \frac{1 - tg^2\dfrac{\alpha}{2}}{1 + tg^2\dfrac{\alpha}{2}}$$

* tali relazioni valgono per ogni angolo β, quindi anche per $\beta = \frac{\alpha}{2}$