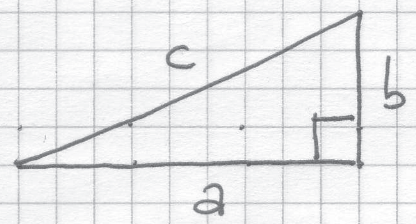
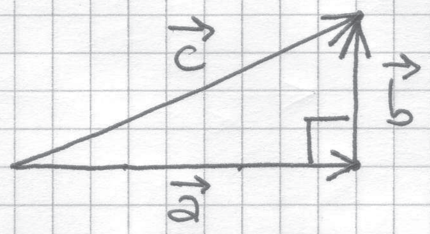
TEOREMA DI PITAGORA

dimostrato col calcolo vettoriale



Vettorializzando i 3 lati in modo che :

$\vec{c} = \vec{a} + \vec{b}$



ricordando che il prodotto scalare $\bullet$ e' nullo allorche' i 2 vettori sono $\perp$, si ricava :

$$c^2 = |\vec{c}|^2 = \vec{c} \bullet \vec{c} = (\vec{a} + \vec{b}) \bullet (\vec{a} + \vec{b}) =$$

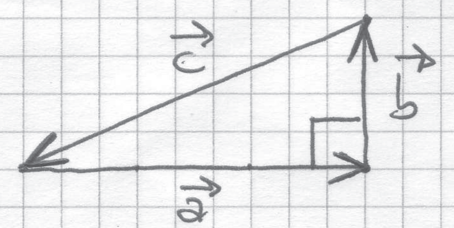
$$= \vec{a} \bullet \vec{a} + \vec{a} \bullet \vec{b} + \vec{b} \bullet \vec{a} + \vec{b} \bullet \vec{b} =$$

$$= |\vec{a}|^2 + 0 + 0 + |\vec{b}|^2 = a^2 + b^2 \quad ;$$

se invece vettorializziamo i 3 lati in modo che

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$ cioe'

$\vec{c} = -(\vec{a} + \vec{b})$



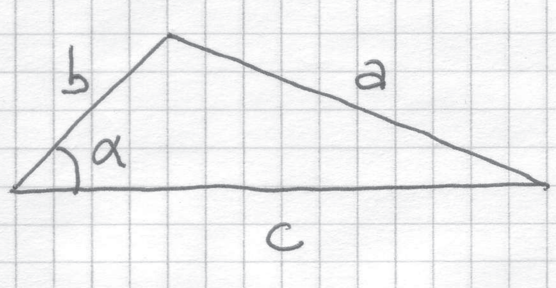
si ottiene $c^2 = |\vec{c}|^2 = \vec{c} \bullet \vec{c} =$

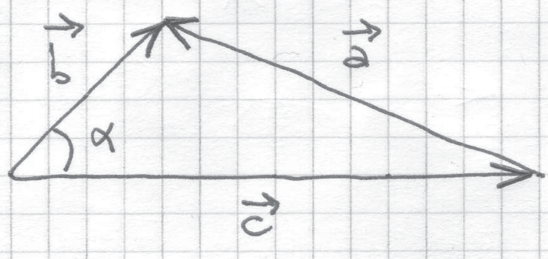
$$= [-(\vec{a} + \vec{b})] \bullet [-(\vec{a} + \vec{b})] = (\vec{a} + \vec{b}) \bullet (\vec{a} + \vec{b}) =$$

$$= \ldots = a^2 + b^2$$

TEOREMA DEL COSENO (o di Carnot o legge dei coseni)

dimostrato col calcolo vettoriale





se $\vec{b} = \vec{c} + \vec{a}$

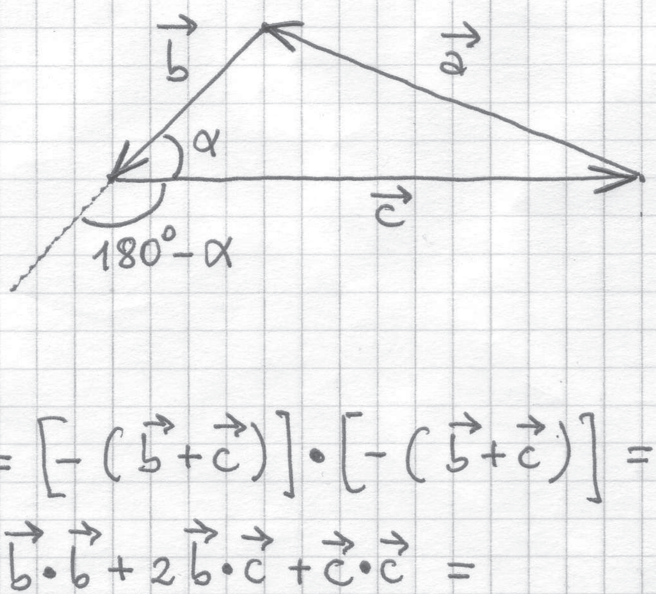
cioe' $\vec{a} = \vec{b} - \vec{c}$

allora :

$$a^2 = |\vec{a}|^2 = \vec{a} \cdot \vec{a} = (\vec{b} - \vec{c}) \cdot (\vec{b} - \vec{c}) =$$

$$= \vec{b} \cdot \vec{b} - \vec{b} \cdot \vec{c} - \vec{c} \cdot \vec{b} + \vec{c} \cdot \vec{c} = \vec{b} \cdot \vec{b} - 2\,\vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{c} =$$

$$= |\vec{b}|^2 - 2|\vec{b}||\vec{c}| \cos \alpha + |\vec{c}|^2 = b^2 + c^2 - 2bc \cos \alpha \quad ;$$



se invece

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$ cioe'

$\vec{a} = -(\vec{b} + \vec{c})$

allora :

$$a^2 = |\vec{a}|^2 = \vec{a} \cdot \vec{a} = \left[-(\vec{b} + \vec{c})\right] \cdot \left[-(\vec{b} + \vec{c})\right] =$$

$$= (\vec{b} + \vec{c}) \cdot (\vec{b} + \vec{c}) = \vec{b} \cdot \vec{b} + 2\,\vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{c} =$$

$$= |\vec{b}|^2 + 2|\vec{b}||\vec{c}| \cos(180° - \alpha) + |\vec{c}|^2 =$$

$$= b^2 + c^2 - 2bc \cos \alpha$$