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$$\int_0^{\frac{\sqrt{2}}{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx =$$

Sostituzione $x = \operatorname{sen} t \rightarrow t = \arcsin x$

$dx = \cos t\, dt \qquad \sqrt{1-x^2} = \sqrt{1-\operatorname{sen}^2 t} = \sqrt{\cos^2 t} =$
$= \cos t$

estremi: per $x = 0 \qquad t = 0$
per $x = \frac{\sqrt{2}}{2} \qquad t = \frac{\pi}{4}$

$$= \int_0^{\frac{\pi}{4}} \frac{\operatorname{sen}^2 t}{\sqrt{1-\operatorname{sen}^2 t}} \cdot \cos t\, dt =$$

$$= \int_0^{\frac{\pi}{4}} \frac{\operatorname{sen}^2 t}{\cancel{\cos t}} \cdot \cancel{\cos t}\, dt =$$

$$= \int_0^{\frac{\pi}{4}} \operatorname{sen}^2 t\, dt =$$

dalle formule di duplicazione

$\cos 2x = \cos^2 x - \operatorname{sen}^2 x$

$\cos 2x = 1 - \operatorname{sen}^2 x - \operatorname{sen}^2 x$

$\cos 2x = 1 - 2\operatorname{sen}^2 x$

$\downarrow$

$\operatorname{sen}^2 x = \frac{1 - \cos 2x}{2}$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{2}\, dt - \frac{1}{2}\int_0^{\frac{\pi}{4}} \cos 2t \,\frac{2}{2}\, dt =$$

$$= \left[\frac{1}{2}t\right]_0^{\frac{\pi}{4}} - \frac{1}{2}\left[\frac{1}{2}\operatorname{sen} 2t\right]_0^{\frac{\pi}{4}} =$$

$$= \frac{\pi}{8} - \frac{1}{4}$$