Formule Goniometriche:

Formule di ADDIZIONE E SOTTRAZIONE:

$$\cos(\alpha \pm \beta) = \cos\alpha \cdot \cos\beta \mp \operatorname{sen}\alpha \cdot \operatorname{sen}\beta$$
$$\operatorname{sen}(\alpha \pm \beta) = \operatorname{sen}\alpha \cdot \cos\beta \pm \cos\alpha \cdot \operatorname{sen}\beta$$
$$tg(\alpha \pm \beta) = \frac{tg\,\alpha \pm tg\,\beta}{1 \mp tg\,\alpha \cdot tg\,\beta}$$

dim. sul libro

dalle formule sugli archi associati: $\operatorname{sen} x = \cos(90° - x)$ considerando l'angolo $\alpha + \beta$

$$\operatorname{sen}(\alpha+\beta) = \cos[90° - (\alpha+\beta)] = \cos[(90°-\alpha)+\beta] = \cos(90°-\alpha)\cos\beta + \operatorname{sen}(90°-\alpha)\operatorname{sen}\beta$$
$$= \operatorname{sen}\alpha\cos\beta + \cos\alpha\operatorname{sen}\beta$$

$$tg(\alpha+\beta) = \frac{\operatorname{sen}(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\operatorname{sen}\alpha\cdot\cos\beta + \cos\alpha\cdot\operatorname{sen}\beta}{\cos\alpha\cdot\cos\beta - \operatorname{sen}\alpha\cdot\operatorname{sen}\beta} = \frac{\frac{\operatorname{sen}\alpha\cdot\cancel{\cos\beta}}{\cos\alpha\cdot\cancel{\cos\beta}} + \frac{\cancel{\cos\alpha}\cdot\operatorname{sen}\beta}{\cancel{\cos\alpha}\cdot\cos\beta}}{\frac{\cancel{\cos\alpha\cdot\cos\beta}}{\cancel{\cos\alpha\cdot\cos\beta}} - \frac{\operatorname{sen}\alpha\cdot\operatorname{sen}\beta}{\cos\alpha\cdot\cos\beta}} = \frac{tg\,\alpha + tg\,\beta}{1 - tg\,\alpha\cdot tg\,\beta}$$

Formule di DUPLICAZIONE:

$$\operatorname{sen}2\alpha = 2\operatorname{sen}\alpha\cdot\cos\alpha$$
$$\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha$$
$$tg\,2\alpha = \frac{2tg\,\alpha}{1-tg^2\alpha}$$

Dim.: $\operatorname{sen}2\alpha = \operatorname{sen}(\alpha+\alpha) = \operatorname{sen}\alpha\cdot\cos\alpha + \cos\alpha\cdot\operatorname{sen}\alpha = 2\operatorname{sen}\alpha\cdot\cos\alpha$

Dim.: $\cos 2\alpha = \cos(\alpha+\alpha) = \cos\alpha\cdot\cos\alpha - \operatorname{sen}\alpha\cdot\operatorname{sen}\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha$

Dim.: $tg\,2\alpha = tg(\alpha+\alpha) = \frac{tg\,\alpha + tg\,\alpha}{1 - tg\alpha\cdot tg\,\alpha} = \frac{2tg\,\alpha}{1-tg^2\alpha}$

Formule di BISEZIONE:

$$\operatorname{sen}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

Dim.: $\cos 2\beta = \cos^2\beta - \operatorname{sen}^2\beta = 1 - \operatorname{sen}^2\beta - \operatorname{sen}^2\beta = 1 - 2\operatorname{sen}^2\beta$ ➔ $2\operatorname{sen}^2\beta = 1 - \cos 2\beta$

➔ $\operatorname{sen}\beta = \pm\sqrt{\frac{1-\cos 2\beta}{2}}$ ➔* $\operatorname{sen}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

Dim.: $\cos 2\beta = \cos^2\beta - \operatorname{sen}^2\beta = \cos^2\beta - (1-\cos^2\beta) = 2\cos^2\beta - 1$ ➔ $2\cos^2\beta = 1 + \cos 2\beta$

➔ $\cos\beta = \pm\sqrt{\frac{1+\cos 2\beta}{2}}$ ➔* $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

$$tg\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

Dim.: $tg\frac{\alpha}{2} = \frac{\operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{\pm\sqrt{\frac{1-\cos\alpha}{2}}}{\pm\sqrt{\frac{1+\cos\alpha}{2}}} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

$$tg\frac{\alpha}{2} = \frac{\operatorname{sen}\alpha}{1+\cos\alpha}$$

Dim.: $tg\frac{\alpha}{2} = \frac{\operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{\operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\cdot\frac{2\cos\frac{\alpha}{2}}{2\cos\frac{\alpha}{2}} = \frac{2\operatorname{sen}\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}} = \frac{\operatorname{sen}\alpha}{\cancel{2}\frac{1+\cos\alpha}{\cancel{2}}} = \frac{\operatorname{sen}\alpha}{1+\cos\alpha}$

$$tg\frac{\alpha}{2} = \frac{1-\cos\alpha}{\operatorname{sen}\alpha}$$

Dim.: $tg\frac{\alpha}{2} = \frac{\operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{\operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\cdot\frac{2\operatorname{sen}\frac{\alpha}{2}}{2\operatorname{sen}\frac{\alpha}{2}} = \frac{2\operatorname{sen}^2\frac{\alpha}{2}}{2\operatorname{sen}\frac{\alpha}{2}\cos\frac{\alpha}{2}} = \frac{\cancel{2}\frac{1-\cos\alpha}{\cancel{2}}}{\operatorname{sen}\alpha} = \frac{1-\cos\alpha}{\operatorname{sen}\alpha}$

Formule PARAMETRICHE:

$$\operatorname{sen}\alpha = \frac{2tg\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$$

$$\cos\alpha = \frac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$$

Dim.: $\operatorname{sen}2\beta = \frac{2\operatorname{sen}\beta\cdot\cos\beta}{1} = \frac{2\operatorname{sen}\beta\cdot\cos\beta}{\cos^2\beta + \operatorname{sen}^2\beta} = \frac{\frac{2\operatorname{sen}\beta\cdot\cancel{\cos\beta}}{\cos^{\cancel{2}}\beta}}{\frac{\cos^2\beta}{\cos^2\beta} + \frac{\operatorname{sen}^2\beta}{\cos^2\beta}} = \frac{2tg\,\beta}{1+tg^2\beta}$ ➔* $\operatorname{sen}\alpha = \frac{2tg\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$

Dim.: $\cos 2\beta = \frac{\cos^2\beta - \operatorname{sen}^2\beta}{1} = \frac{\cos^2\beta - \operatorname{sen}^2\beta}{\cos^2\beta + \operatorname{sen}^2\beta} = \frac{\frac{\cos^2\beta}{\cos^2\beta} - \frac{\operatorname{sen}^2\beta}{\cos^2\beta}}{\frac{\cos^2\beta}{\cos^2\beta} + \frac{\operatorname{sen}^2\beta}{\cos^2\beta}} = \frac{1-tg^2\beta}{1+tg^2\beta}$ ➔* $\cos\alpha = \frac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$

* tali relazioni valgono per ogni angolo β, quindi anche per $\beta = \frac{\alpha}{2}$